Levene's Test

How do we determine whether two populations have the same variance? The Levene's test (also referred to as the F-test of sample variance) attempts to answer this. To do so, the Levene's test considers the following hypotheses:

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

The main principle of the Levene's test is to contrast the variability between the groups (numerator of the test statistic) with the variability within the groups (denominator): The higher the share of the between-group variation, the higher the value of the test statistic and, thus, the higher the tendency to reject H_0. The exact test statistic F is calculated using the following formula:

$$F = (n-2)\frac{\sum_{j=1}^{2} n_j (\bar{y}_j - \bar{y})^2}{\sum_{j=1}^{2}\sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2}$$

with

$$y_{ij} = |x_{ij} - \bar{x}_j| \text{ and } \bar{x}_j = \frac{1}{n_j}\Sigma_{i=1}^{n_j} x_{ij},$$

$$\bar{y}_j = \Sigma_{i=1}^{n_j} y_{ij} \text{ and } \bar{y} = \frac{1}{2}\Sigma_{j=1}^{2} \bar{y}_j,$$

where n is the total number of observations and n_j the sample size of group j.

The null hypothesis is rejected if F exceeds the $\left(1-\frac{\alpha}{2}\right)$ -quantile of an F(1, n-2)-distribution. We won't discuss the F-distribution in detail - you just need to understand how to read the tabulated quantiles of an F-distribution determined by the significance level a (e.g. $\alpha = 5\%$), the number of groups (here = 2), as well as the number of observations n (here = 20).

To answer our research question on whether the point of sale display and the free tasting stand lead to the same sales volume (Chapter 6), we first have to determine whether both campaign types exhibit the same variability in sales. Consequently, we need to ascertain the different components and auxiliary statistics of F:

The total number observations n is 20, with 10 originating from each campaign type (n_1=10 and n_2=10). The mean sales of the point of sale display is $\bar{x}_1 = 47.30$, whereas the mean sales of the free-tasting stand is $\bar{x}_2 = 52.00$. Table A6.1 illustrates the absolute values of the differences between the single observations and their respective group means:

Table A6.1 Differences in observations

Point of sale display: $y_{1j} = \lvert x_{i1} - \bar{x}_1 \rvert$	Free tasting stand: $y_{2j} = \lvert x_{i2} - \bar{x}_2 \rvert$
2.7	3.0
4.7	3.0
4.3	3.0
0.7	5.0
0.3	3.0
2.3	3.0
3.3	4.0
1.7	2.0
3.7	2.0
3.3	8.0

Based on this tabulation, we arrive at group means $\bar{Y}_1 = 2.70$ and $\bar{Y}_2 = 3.60$ respectively by dividing the sum of the values of the two columns (27 and 36) by the number of entries per column ($n_1 = n_2 = 10$). Based on these results, the test statistic can be calculated and compared to the $\left(1-\frac{\alpha}{2}\right)$-quantile of an F(1, 28)-distribution.

$$F = 18 \cdot \frac{10 \cdot (0.45^2 + 0.45^2)}{19.20 + 28.40} = \frac{72.90}{47.60} = 1.5315126(\ldots) \approx 1.532$$

Using a significance level of $\alpha = 5\%$, the respective F(1, 28)-quantile amounts to about 5.61, which exceeds the value of the test statistic F. Consequently, we cannot reject H_0. Thus, we have to revert to the pooled variance estimate when comparing the two groups, which is done in Chapter 6 of the book.

Note that the Levene's test can also be applied to check whether the variances of more than two groups are equal or not.[1]

[1] For k groups: $H_0: \sigma_1^2 = \cdots = \sigma_k^2$ vs. $H_1: \sigma_i^2 \neq \sigma_j^2$ for at least two groups i and j with i $\neq$ j Test statistic:

$$F = \frac{n-k}{k-1} \frac{\sum_{j=1}^{k} n_j (\bar{y}_j - \bar{y})^2}{\sum_{j=1}^{k} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2}$$

Rejection region:

$$F > F^{1-\frac{\alpha}{2}}(k-l, n-k)$$